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MODEL WITH A CONTINUUM OF GOODS
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COMPARATIVE ADVANTAGE, TRADE AND PAYMENTS IN A RICARDIAN MODEL WITH A CONTINUUM OF GOODS*

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This paper discusses Ricardian trade and payments theory in the case of a continuum of commodities. The analysis thus extends the development of the many-commodity, two-country comparative advantage analysis as presented, for example in Haberler (1937) and as historically reviewed by Chipman (1965). Perhaps surprisingly, the continuum assumption simplifies the analysis in comparison with the discrete many-commodity case. The distinguishing feature of the Ricardian approach emphasized in this paper is the determination of the competitive margin in production between imported and exported goods. The analysis advances the existing literature by showing formally how tariffs and transport costs establish a range of commodities that are not traded, and how the price-specie flow mechanism does or does not give rise to movements in relative cost and price levels.

The formal real model is introduced in Part I. Its equilibrium determines the relative wage and price structure and the efficient international specialization pattern. Part II considers standard comparative static questions of growth, demand shifts, technological change, and transfers. Extensions of the model to nontraded goods, tariffs and

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transport costs are then studied in Part III. Monetary considerations are introduced in Part IV, which examines the price-specie mechanism under stable parities, floating exchange rate regimes, and also questions of unemployment under sticky money wages.
I. THE REAL MODEL

In this part we develop the basic real model and determine the equilibrium relative wage and price structure along with the efficient geographic pattern of specialization. Assumptions about technology are specified in Section A. Section B deals with demand. In Section C the equilibrium is constructed and some of its properties are explored. Throughout this section we assume zero transport costs and no other impediments to trade.

A. Technology and Efficient Geographic Specialization

The many-commodity Ricardian model assumes constant unit labor requirements \((a_1, \ldots, a_n)\) and \((a_1^*, \ldots, a_n^*)\) for the \(n\) commodities that can be produced in the home and foreign countries, respectively.\(^1\) The commodities are conveniently indexed so that relative unit labor requirements are ranked in order of diminishing home-country comparative advantage,

\[
a_1^*/a_1 > \ldots > a_i^*/a_i > \ldots > a_n^*/a_n
\]

where an asterisk denotes the foreign country.

In working with a continuum of goods, we similarly index commodities on an interval, say \([0,1]\), in accordance with diminishing home-country comparative advantage. A commodity, \(z\), is associated with each point on the interval, and for each commodity there are unit labor requirements in the two countries, \(a(z)\) and \(a^*(z)\), with relative unit labor requirement given by

\[
A(z) = \frac{a^*(z)}{a(z)}; \quad A'(z) < 0.
\]

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\(^1\)For a review of the Ricardian model, see Chipman (1965).
The relative unit labor requirement function in (1) is by assumption continuous and by construction (ranking or indexing of goods) non-increasing in \( z \). We make the stronger assumption that \( A(z) \) is in fact decreasing in \( z \). The function \( A(z) \) is shown in Figure 1 as the downward sloping schedule.

Consider now the range of commodities produced domestically and those produced abroad, as well as the relative price structure associated with given wages. For that purpose we define as \( w \) and \( w^* \) the domestic and foreign wages measured in any (common!) unit. The home country will efficiently produce all those commodities for which domestic unit labor costs are less than or equal to foreign unit labor costs. Accordingly, any commodity \( z \) will be produced at home if

\[
(2) \quad a(z)w \leq a^*(z)w^*
\]

or

\[
(2') \quad w \leq A(z)
\]

where

\[
(3) \quad \omega \equiv w/w^*
\]

is the ratio of our real wage to theirs (our "double-factorial terms of trade"). It follows that for a given relative wage \( \omega \) the home country will efficiently produce the range of commodities

\[
(4) \quad 0 \leq z \leq \tilde{z}(\omega),
\]

where, taking \((2')\) with equality defines the borderline commodity

\[
(5) \quad \tilde{z} = A^{-1}(\omega),
\]

\( A^{-1}(\ ) \) being the inverse function of \( A(\ ) \).
By the same argument the foreign country will specialize in the production of commodities in the range

\[ \tilde{z}(\omega) \leq z \leq 1. \]

The minimum cost condition determines the structure of relative prices. The relative price of a commodity \( z \) in terms of any other commodity \( z' \), with both goods produced in the home country, is equal to the ratio of home unit labor costs:

\[ \frac{P(z)}{P(z')} = \frac{wa(z)}{wa(z')} = \frac{a(z)}{a(z')} ; \ z \leq \tilde{z}, \ z' \leq \tilde{z}. \]

The relative price of home-produced \( z \) in terms of a commodity \( z'' \) produced abroad by contrast is

\[ \frac{P(z)}{P(z'')} = \frac{wa(z)}{wa(z'')} = \frac{a(z)}{a(z'')} ; \ z < \tilde{z} < z''. \]

In summarizing the supply part of the model we note that any specified relative real wage is associated with an efficient geographic specialization pattern characterized by the borderline commodity \( \tilde{z}(\omega) \) as well as a relative price structure. (The pattern is "efficient" in the sense that the world is on, and not inside, its production-possibility frontier.)

**B. Demand**

On the demand side we impose a strong homothetic structure in the form of J.S. Mill or Cobb-Douglas demand functions that associate with each commodity (i) a constant expenditure share, \( b_i \). We further assume identical tastes for the two countries or uniform homothetic demand.
By analogy with the many commodity case, which involves budget shares
\[ b_i = \frac{P_i C_i}{Y} ; \quad \sum_{i} b_i = l ; \quad b_i = b_i^* \]
we therefore have for the continuum case a given \( b(z) \) profile:

\[ b(z) = \frac{P(z) C(z)}{Y} > 0 ; \quad \int_{0}^{l} b(z) dz = 1 ; \quad b(z) = b^*(z) \]

where \( Y \) denotes income, \( C \) demand for and \( P \) the price of commodity \( z \).

Next we define the fraction of income spent (anywhere) on those goods in which the home country has a comparative advantage:

\[ \theta(\tilde{z}) = \int_{0}^{\tilde{z}} b(z) dz ; \quad \theta'(\tilde{z}) = b(\tilde{z}) > 0 \]

where again \((0,\tilde{z})\) denotes the range of commodities for which the home country enjoys a comparative advantage. With a fraction \( \theta \) of each country's income, and therefore of world income, spent on domestically produced goods it follows that the fraction of income spent on foreign produced commodities is

\[ 1 - \theta(\tilde{z}) = \int_{\tilde{z}}^{l} b(z) dz. \]

C. Equilibrium Relative Wages and Specialization

To derive the equilibrium relative wage and price structure and the associated pattern of efficient geographic specialization we turn next to the condition of market equilibrium. Consider the home country's labor market, or equivalently the market for domestically produced commodities.
With \( \tilde{z} \) denoting the hypothetical dividing line between domestically and foreign produced commodities, equilibrium in the market for home produced goods requires that domestic income, \( wL \), equals world spending on domestically produced goods:

\[
(10) \quad wL = \theta(\tilde{z})(wL + w^*L^*).
\]

Equation (10) associates with each \( \tilde{z} \) a value of the relative wage \( w/w^* \) such that market equilibrium obtains. This schedule is drawn in Figure 1 as the upward sloping locus and is obtained from (10) by rewriting the equation in the form:

\[
(10') \quad \omega = \frac{\theta(\tilde{z})}{1 - \theta(\tilde{z})} (L^*/L) = B(\tilde{z}; L^*/L)
\]

where it is apparent from (9) that the schedule starts at zero and approaches infinity as \( \tilde{z} \) approaches unity. To interpret the \( B(\ ) \) schedule we note that it is entirely a representation of the demand side and in that respect shows that if the range of domestically produced goods were increased, at constant relative wages, demand for domestic labor (goods) would increase as the dividing line is shifted, at the same time that demand for foreign labor (goods) would decline.\(^1\) A rise in the domestic relative wage is required to equate the demand for domestic labor to the existing supply.

An alternative interpretation of the \( B(\ ) \) schedule as the locus of trade balance equilibria uses the fact that (10) can be written in the

---

\(^1\) Throughout this paper we refer to "domestic" goods as commodities produced in the home country rather than to commodities that are non-traded. The latter we call "nontraded" goods.
FIGURE 1
form:

\[(10') \quad [1-\theta(\tilde{z})]wL = \theta(\tilde{z})w^*L^*\]

This states that equilibrium in the trade balance means imports are equal in value to exports. On this interpretation, the \(B(\ )\) schedule is upward sloping because an increase in the range of commodities hypothetically produced at home at constant relative wages lowers our imports and raises our exports. The resulting trade imbalance must be corrected by an increase in our relative wage that will raise our import demand for goods produced abroad to our exports and thus restore balance.

The next step is to combine the demand side of the economy with the condition of efficient specialization as represented in equation (5), which specifies the competitive margin as a function of the relative wage. Substituting (5) in (10') yields as a solution the unique relative wage, \(\tilde{w}\), at which the world is efficiently specialized, is in balanced trade, and at full-employment with all markets clearing:

\[(11) \quad \tilde{w} = A(\tilde{z}) = B(\tilde{z}; L^*/L).\]

The equilibrium relative wage defined in (11) is represented in Figure 1 at the intersection of the \(A(\ )\) and \(B(\ )\) schedules. Commodity \(\tilde{z}\) denotes the equilibrium borderline of comparative advantage between commodities produced and exported by the home country \((0 \leq z \leq \tilde{z})\) and those commodities produced and exported by the foreign country \((\tilde{z} \leq z \leq 1)\).

Among the characteristics of the equilibrium we note that the equilibrium relative wage and specialization pattern are determined by technology, tastes and relative size as measured by the relative labor

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\footnote{See Appendix I for the relation of the diagram to previous analyses.}
The relative price structure associated with the equilibrium at point E is defined by equations (6) and (7), yielding the relative wage \( \bar{\omega} \) and the equilibrium specialization pattern \( z(\bar{\omega}) \).

The equilibrium levels of production, \( Q(z) \) and \( Q^*(z) \), and employment in each industry, \( L(z) \) and \( L^*(z) \), can be recovered from the demand structure, unit labor requirements, and the comparative advantage pattern:

\[
Q(z) = \frac{b(z)}{a(z)} (L + \bar{\omega}^{-1}L^*) = C(z) + C^*(z) \quad , \quad 0 \leq z \leq \bar{z}
\]

\[
= 0 \quad , \quad \bar{z} \leq z < 1
\]

\[
Q^*(z) = \frac{b(z)}{a^*(z)} (\bar{\omega}L + L^*) = C(z) + C^*(z) \quad , \quad \bar{z} < z \leq 1
\]

\[
= 0 \quad , \quad 0 \leq z \leq \bar{z}
\]

\[
L(z) = b(z)(L + \bar{\omega}^{-1}L^*) \quad , \quad 0 \leq z < \bar{z}
\]

\[
= 0 \quad , \quad \bar{z} \leq z < 1
\]

\[
L^*(z) = b(z)(\bar{\omega}L + L^*) \quad , \quad \bar{z} < z \leq 1
\]

\[
= 0 \quad , \quad 0 \leq z < \bar{z}
\]

\[
\bar{\omega}L(z) + L^*(z) = b(z)(\bar{\omega}L + L^*)
\]

\[
Q(z) + Q^*(z) = C(z) + C^*(z) .
\]

---

2 The construction of the \( B(\cdot) \) schedule relies heavily on the Cobb-Douglas demand structure. If, instead, demand functions were identical across countries and homothetic, an analogous schedule could be constructed. In the general homothetic case, however, a set of relative prices is required at each \( z \) to calculate the equivalent of the \( B(\cdot) \) schedule; the relative prices are those that apply on the \( A(\cdot) \) schedule for that value of \( z \). In this case the independence of the \( A(\cdot) \) and \( B(\cdot) \) schedules is obviously lost. In the general homothetic case there is still a unique intersection of the
We note that with identical homothetic tastes across countries and no distortions, the relative wage \( \bar{w} \) is in fact a measure of the well-being of the representative person-laborer at home relative to the well-being of the representative foreign laborer.

\(^2\) (continued from preceding page) A(z) and B( ) schedules. For more general non-homothetic demand structures, it is known that even in the case of two Ricardian goods there may be no unique equilibrium though there will almost always be a finite number of equilibria. See Debreu (1970) and Smale (1966).
II. COMPARATIVE STATICS

The unique real equilibrium in Figure 1 is determined jointly by tastes, technology and relative size, \( L^* / L \). We can now exploit Figure 1 to examine simple comparative static questions.

A. Relative Size

Consider first the effect of an increase in the relative size of the rest of the world. An increase in \( L^* / L \) by (5) shifts the \( B(\_\_\_\_) \) trade balance equilibrium schedule upward in proportion to the change in relative size and must, therefore, raise the equilibrium relative wage at home and reduce the range of commodities produced domestically. It is apparent from Figure 2 that the domestic relative wage increases proportionally less than the decline in domestic relative size.

The rise in equilibrium relative wages due to a change in relative size can be thought of in the following manner. At the initial equilibrium, the increase in the foreign relative labor force would create an excess supply of labor abroad and an excess demand for labor at home, or, correspondingly, a trade surplus for the home country. The resulting increase in domestic relative wages serves to eliminate the trade surplus while at the same time raising relative unit labor costs at home. The increase in domestic relative unit labor costs in turn implies a loss of comparative advantage in marginal industries and thus a reduction in the range of commodities produced domestically.

The welfare implications of the change in relative size take the form of an unambiguous improvement in the home country's real income and a reduction in real income per head abroad. We observe, too, that from the definition of the home country's share in world income and (10) we have
FIGURE 2
\[ \frac{wL}{(wL + wL^*)} = 0 \]

It is apparent therefore that a reduction in domestic relative size in raising the domestic relative wage and thereby reducing the range of commodities produced domestically, must under our Cobb-Douglas demand assumptions lower the home country's share in total world income and spending -- even though its per capita income rises.

B. Technical Progress

In this section we will be concerned with the effects of uniform technical progress. By equation (1), a uniform proportional reduction in foreign unit labor requirements implies a reduction in \( \alpha(z) \) and therefore a downward shift of the \( A(z) \) schedule in Figure 1. At the initial relative wage, \( \bar{w} \), the loss of our comparative advantage due to a reduction in foreign unit labor costs will imply a loss of some industries in the home country and a corresponding trade deficit. The resulting induced decline in the equilibrium relative wage serves to restore trade balance equilibrium and to offset, in part, our decline in comparative advantage.

The net effect is, therefore, a reduction in domestic relative wages that must fall proportionally short of the decline in relative unit labor requirements abroad. The home country's terms of trade therefore improve as can be noted by using (7) for any two commodities \( z \) and \( z'' \), respectively, produced at home and abroad.

\[ \hat{P}(z) - \hat{P}(z'') = \hat{\bar{w}} - \hat{\alpha}^{*}(z'') > 0 \]

where \( \hat{\alpha} \) denotes a proportional change. Furthermore, domestic real income
increases, as does foreign real income. The range of goods produced domestically declines since domestic labor, the efficiency units, is now relatively more scarce.

An alternative form of technical progress that can be studied is the international transfer of the least cost technology. Such transfers reduce the discrepancies in relative unit labor requirements -- by lowering them for each $z$ in the relatively less efficient country -- and therefore flatten the $A(z)$ schedule in Figure 1. It can be shown that such harmonization of technology must benefit the innovating low-wage country, and that it may reduce real income in the high-wage country whose technology comes to be adopted. In fact, the high-wage country must lose if harmonization is complete so that relative unit labor requirements now become identical across countries.

---

1 The purchasing power of foreign labor income in terms of domestically produced goods is $w^* L^*/wa(z) = L^*/a(z)\bar{\omega}$ and in terms of foreign goods $L^*/a^*(z)$. The fact that foreigners' real income per head rises is guaranteed by our Cobb-Douglas demand assumption. In the general homothetic case, a balanced reduction in $a^*(z)$ can be immiserizing abroad as the real wage falls strongly in terms of all previously imported goods; the drop in $a^*(z)$ in the general homothetic case always increases our real wage.

2 Complete equalization of unit labor requirements implies that the $A(z)$ schedule is horizontal at the level $\omega = A(z) = 1$. In this case geographic specialization becomes indeterminate and inessential.
C. Demand Shifts

The case with a continuum of commodities requires a careful definition of a demand shift. For our purposes it is sufficient to ask, What is the effect of a shift from high z commodities toward low z commodities? It is apparent from Figure 2 that such a shift will cause the trade balance equilibrium schedule B( ) to shift up and to the left. It follows that the equilibrium domestic relative wage will rise while the range of commodities produced by the home country declines. Domestic labor is allocated to a narrower range of commodities that are consumed with higher density while foreign labor is spread more thinly across a larger range of goods.

Welfare changes cannot be identified in this instance because tastes themselves have changed. It is true that domestic relative income rises along with the relative wage. Further we note that since \( \bar{w} \) rises, the relative well-being of home labor to foreign labor (reckoned at the new tastes) is greater than was our laborers' relative well-being (reckoned at the old tastes).

D. Unilateral Transfers

Suppose foreigners make a continual unilateral transfer to us. With uniform homothetic tastes and no impediments to trade, neither curve is shifted by the transfer since we spend the transfer exactly as foreigners would have spent it but for the transfer. The new equilibrium involves a recurring trade deficit for us, equal to the transfer, but there is no change in the terms of trade. As Ohlin argued against Keynes, here is a case where full equilibration takes place solely as a result of the spending
transfers. When we introduce nontraded goods below, Ohlin's presumption will be found to require detailed qualifications, as it also would if tastes differ geographically.
III. EXTENSIONS OF THE REAL MODEL

Extensions of the real model taken up in this part concern nontraded goods, tariffs and transport costs. The purpose of this section is two-fold. First we establish how the exogenous introduction of nontraded goods qualifies the preceding analysis. Next we turn to a particular specification of tariffs and transport costs to establish an equilibrium range of endogenously determined nontraded goods as part of the equilibrium solution of the model. Transfers are then shown to affect the equilibrium relative price structure and the range of goods traded.

A. Nontraded Goods

To introduce nontraded goods into the analysis we assume that a fraction \( k \) of income is everywhere spent on internationally traded goods and a fraction \( (1-k) \) is spent in each country on nontraded commodities. With \( b(z) \) continuing to denote expenditure densities for traded goods we have accordingly

\[
(14) \quad k = \int_{0}^{1} b(z)dz < 1
\]

where \( z \) denotes traded goods. As before the fraction of income spent on domestically exportable commodities is \( \Theta(z) \), except that \( \Theta \) now reaches a maximum value of \( \Theta(1) = k \).

Equation (1) remains valid for traded goods, but the trade balance equilibrium condition in (10") must now be modified to:

\[
(15) \quad [1-\Theta(z) - (1-k)]wL = \Theta(z)^*w^*L^*
\]

---

\(^1\)We can think of the range of nontraded goods as another \([0,1]\) interval with commodities denoted by \( x \) and expenditure fractions on the goods given by \( c(x) \). With those definitions we have \( \int_{0}^{1} c(x)dx \equiv 1-k. \)
since domestic spending on imports is equal to income less spending on all domestically produced goods including nontraded commodities. Equation (15) can be rewritten as

\[(15'') \quad \omega = \frac{\theta(z)}{k - \theta(\tilde{z})} (L^*/L)\]

where k is a constant and therefore is independent of the relative wage structure.

We note that (15'') together with (5) determines the equilibrium relative wage and efficient geographic specialization, \((\bar{\omega}, \tilde{z})\). Further it is apparent that (15'') has exactly the same properties as (10') and that accordingly a construction of equilibrium like that in Figure 1 remains appropriate. The equilibrium relative wage again depends on relative size, technology and demand conditions. In this case demand conditions explicitly include the fraction of income spent on traded goods:

\[(11') \quad \bar{\omega} = \frac{\theta(z)}{k - \theta(\tilde{z})} \frac{L^*}{L} = A(\tilde{z})\]

This nicely generalizes our previous equilibrium to handle exogenously given nontraded goods.

Two applications of the extended model highlight the special aspects introduced by nontraded goods. First consider a shift in demand (in each country) toward nontraded goods. To determine the effects on the equilibrium relative wage we have to establish whether this shift is at the expense of high or low z commodities. In the former case the home country's relative wage increases while in the latter case it declines.
Consider next a transfer received by the home country in the amount $T$ measured in terms of foreign labor. As is well known, and already shown, with identical homothetic tastes and no nontraded goods, a transfer leaves the terms of trade unaffected. In the present case, however, the condition for balanced trade, inclusive of transfers, becomes:

(16) \[ T = (k - \theta)(\omega L + T) - \theta[L^* - T] \]

or

(16') \[ \omega = \frac{1 - k}{k - \theta(\bar{z})} \frac{(T/L)}{(T/L)} + \frac{\theta(\bar{z})}{k - \theta(\bar{z})} \frac{(L^*)}{(L/L)} \]

It is apparent from (16') that a transfer receipt by the home country causes the trade balance equilibrium schedule in Figure 1 to shift upward at each level of $z$. Accordingly, the equilibrium domestic relative wage increases and the range of commodities produced domestically is reduced. The steps in achieving this result are, first, that at the initial relative wage only a fraction of the transfer is spent on imports in the home country, while foreign demand for domestic goods similarly declines only by a fraction of their reduced income. The resulting surplus for the home country has to be eliminated by, second, an increase in the domestic relative wage and a corresponding improvement in the home country's terms of trade.\(^1\) The analysis of nontraded goods therefore confirms in a Ricardian model the "orthodox" presumption with respect to the terms of trade effects.

---

\(^1\)At constant relative wages the current account worsens by \[ [(1-k-\theta)\theta]dT = (1-k)dT \] which is less than the transfer, since it is equal to the fraction of income spent on nontraded goods.
B. Transport Costs

The notion that transport costs give rise to a range of commodities that are nontraded is established in the literature and is particularly well stated by Haberler (1937). In contrast with the previous section we will now endogenously determine the range of nontraded commodities as part of the equilibrium. We assume, following Samuelson (1954), that transport costs take the form of "shrinkage" in transit so that a fraction g(z) of commodities shipped actually arrives. We further impose the assumption that g = g(z) is identical for all commodities.

The home country will produce commodities for which domestic unit labor cost falls short of foreign unit labor costs adjusted for shrinkage, and we modify (2') accordingly:

\[(17) \quad w_a(z) \leq \frac{1}{g} w_a^*(z) \quad \text{or} \quad \omega \leq A(z)/g \]

Similarly the foreign country produces commodities for which foreign unit labor cost falls short of adjusted unit labor costs of imports:

\[(18) \quad w_a^*(z) \leq \frac{1}{g} w_a(z) \quad \text{or} \quad A(z)g \leq \omega \]

In Figure 3 we show the adjusted relative unit labor requirement schedules A(z)/g and A(z)g. It is apparent from (17) and (18) that for any given relative wage the home country produces and exports commodities to the left of the A(z)g schedule, both countries produce as nontraded

---

1 The pre-Ohlin "orthodox" view of Keynes, Taussig, Viner and other writers is discussed in Viner (1937) and Samuelson (1952, 1954). A recent treatment with nontraded goods is Jones (1975).
FIGURE 3
goods commodities in the intermediate range, and the foreign country produces and exports commodities in the range to the right of $A(z)/g$.

To determine the equilibrium relative wage we turn to the trade balance equilibrium condition in (19) -- together with (20) and (21) -- that is modified to take account of the endogenous range of nontraded goods:

$$ (19) \quad (1-\lambda)L = (1-A^*)L^* $$

The variable $\lambda$ is the fraction of income spent in the home country on domestically produced goods -- exportables and nontraded -- and $\lambda^*$ is the share of foreigners' income spent on goods they produce. Both $\lambda$ and $\lambda^*$ are endogenously determined because the ranges of goods produced in each country depend on the relative wages.

$$ (20) \quad \lambda(gw) \equiv \int_{0}^{\bar{z}} b(z)dz ; \quad \lambda'(gw) < 0 $$

$$ \lambda^*(w/g) \equiv \int_{\bar{z}^*}^{1} b(z)dz ; \quad \lambda^*'(w/g) > 0. $$

The dependence of $\lambda(\ )$ and $\lambda^*(\ )$ on the variables specified in (20) and the respective derivatives follow from (21) below.

The limits of integration, $\bar{z}$ and $\bar{z}^*$, are derived from the conditions for efficient production in (17) and (18) by imposing equalities and so defining the borderline commodities. Thus, $\bar{z}$ is the borderline between domestic nontraded goods and imports for the home country, and $\bar{z}^*$ denotes the borderline between foreign nontraded goods and the home country's
exports:

\[(21) \quad \tilde{z}^* = A^{-1}(\omega/g) \quad ; \quad d\tilde{z}^*/d\omega < 0\]

\[\hat{z} = A^{-1}(g\omega) \quad ; \quad d\hat{z}/d\omega < 0.\]

Of course, equilibrium \(\hat{z}\) and \(\tilde{z}^*\) are yet to be determined by the interaction of technology and demand conditions.

From (21) an increase in the relative wage reduces the range of commodities domestically produced and therefore raises the fraction of income spent on imports. Abroad the converse holds. An increase in the domestic relative wage increases the range of goods produced abroad and therefore reduces the fraction of income spent on imports. It follows that we can solve:

\[(19') \quad \bar{\omega} = \frac{1 - \lambda^*(\bar{\omega}/g)}{1 - \lambda(g\bar{\omega})} \left( \frac{L^*}{L} \right) \equiv \phi(\bar{\omega}; L^*/L, g) \quad ; \quad \frac{\partial \phi}{\partial \bar{\omega}} < 0\]

for the unique equilibrium relative wage as a function of relative size and transport costs:

\[(22) \quad \bar{\omega} = \tilde{\omega}(L^*/L, g).\]

The equilibrium relative wage in (22) in turn in conjunction with (21) determines the equilibrium geographic production pattern, \(\tilde{z}\) and \(\tilde{z}^*\). Since the range of nontraded goods, \(\tilde{z}^* \leq z \leq \hat{z}\), in this formulation depends on the equilibrium relative wage it is obvious that changes in parameters shift that range across commodities. Thus a transfer that raises the equilibrium relative wage at home causes previously exported commodities to become nontraded and previously nontraded commodities to become importables.
C. Tariffs

We consider next the case of zero transport cost but where each
country levies a uniform tariff on imports at the rates $t$ and $t^*$,
respectively, with proceeds rebated in lump sum form. This case, too,
leads to a range of commodities that are not traded, with the boundaries
defined by:

\begin{align}
\bar{z} &= A^{-1} \left( \frac{\omega}{1+t} \right) \\
\bar{z}^* &= A^{-1} (\omega(1+t^*))
\end{align}

The trade balance equilibrium condition, at international prices, becomes
in place of (19):

\begin{equation}
(1-\lambda)Y / (1+t) = (1-\lambda^*)Y^* / (1+t^*)
\end{equation}

where $Y$ and $Y^*$ denote incomes inclusive of lumpsum tariff rebates. Using
the fact that rebates are equal to the tariff rate times the fraction of
income spent on imports, we arrive at the trade balance equilibrium con-
dition in the form:1

\begin{equation}
\omega = \left( \frac{1-\lambda^*}{1-\lambda} \right) \frac{l + t\lambda^*}{1 + t^*} (L^*/L)
\end{equation}

which can be solved for the equilibrium relative wage as a function of
relative size and the tariff structure:

\begin{equation}
\bar{\omega} = \bar{\omega} (L/L, t, t^*).
\end{equation}

---

1Tariff rebates in the home country are equal to $R = (1-\lambda)Yt/(1+t)$. With $Y \equiv WL+R$ we therefore have $Y = WL(1+t)/(1+t\lambda)$ as an expression for income inclusive of transfers.
From (26) and (23) it is apparent now that the range of nontraded goods will be a function of both tariff rates. It is readily shown that an increase in the tariff improves the imposing country's relative wage and terms of trade. Furthermore, as is well-known, for a single "small" tariff there will be a welfare improvement in the tariff imposing country.

A further question suggested by (26) concerns the effect of a uniform increase in world tariffs. Starting from zero a small uniform increase in tariffs raises the relative wage of the country whose commodities command the larger share in world spending. This result occurs for two reasons. First, at the initial relative wage a larger share of spending out of tariff rebates falls on the goods of the country that commands a larger share in world demand. Second, the tariff induces nontraded goods and therefore increases net demand for the borderline commodity of the country whose residents have the larger income, or equivalently, larger share in world income.

If countries are of equal size as measured by the share in world income, such a uniform tariff increase has no effect on relative wages but reduces well-being in both places. Multilateral tariff increases in this case create some nontraded goods and raise the relative price of importables in terms of domestically produced commodities in each country exactly in proportion to the tariff.
IV. MONEY, WAGES AND EXCHANGE RATES

In this part we extend the discussion of the Ricardian model to deal with monetary aspects of trade. Specifically we shall be interested in the determination of exchange rates in a flexible rate system, in the process of adjustment to trade imbalance under fixed rates, and in the role of wage stickiness. The purpose of the extension is to integrate real and monetary aspects of trade.

A. Flexible Exchange Rates

The barter analysis of the preceding parts is readily extended to a world of flexible exchange rates and flexible money wages. Assume a given nominal quantity of money in each country, $M$ and $M^*$ respectively. Further assume constant expenditure velocities $V$ and $V^*$. A flexible exchange rate, and our assumed absence of non-monetary international asset flows, assures trade balance equilibrium and therefore the equality of income and spending in each country. The nominal money supplies and velocities determine nominal income in each country:

\[(27) \quad WL = MV \quad \text{and} \quad W^*L^* = M^*V^* \]

where $W$ and $W^*$ (now in capital letters) denote domestic and foreign money wages in terms of the respective currencies. Further, defining the exchange rate $e$ as the domestic currency price of foreign exchange, the foreign wage measured in terms of domestic currency is $eW^*$ and the relative wage therefore is $\omega \equiv W/eW^*$.

From the definition of the equilibrium real wage ratio, $\bar{\omega}$, we can find an expression for the equilibrium exchange rates:

\[(28) \quad \bar{e} = (1/\bar{\omega}) (\bar{W}/\bar{W}^*) = (1/\bar{\omega}) (MV/M^*V^*) (L^*/L) \]

---

1This is a strong assumption since it makes spending independent of income and non-liquid assets even in the short run.
where (27') defines equilibrium money wages:

\[
\bar{W} = \frac{MV}{L} \quad \text{and} \quad \bar{W}^* = \frac{M^* V^*}{L^*}.
\]

In this simple structure and with wage flexibility, we can separate the determinants of all equilibrium real variables from monetary considerations. Monetary changes or velocity changes in one country will be reflected in equiproportionate changes in prices in that country and in the exchange rate. A real disturbance, as (28) shows, definitely has repercussions on the exchange rate.

Using the results of Part II, we see that an increase in the foreign relative labor force causes a depreciation in the home country's exchange rate as does uniform technical progress abroad. A shift in demand toward domestic goods, by contrast, leads to an appreciation of the exchange rate. A rise in foreign tariffs will cause our currency to depreciate. Each of these real shifts is assumed to take place while \((M, M^*)\) are unchanged and on the simplifying proviso that real income changes leave \(V\) and \(V^*\) unchanged. (Later we examine effects on floating rates when countries will do temporary lending.)

B. Fixed Exchange Rates

In the fixed exchange rates case we assume currencies are fully convertible at a rate pegged by the monetary authorities. In the absence of capital flows and sterilization policy, a trade imbalance is reflected in monetary flows. Specifically the world money supply is redistributed toward the surplus country at precisely the rate of the trade surplus. We assume that the world money supply is given and equal to \(\bar{G}\) measured in terms of domestic currency. The rate of increase of
the domestic quantity of money is therefore equal to the reduction in foreign money valued at the fixed exchange rate $\bar{e}$:

\[ M^* = -e^* M. \]

For a fixed rate world we have to determine in addition to the real variables, $\bar{w}$ and $\bar{z}$, the levels of money wages $W$ and $W^*$ as well as the equilibrium balance of payments associated with each short-run equilibrium. In the long run the balance of payments will be zero as money is redistributed internationally to the point where income equals spending in each country. In the short run an initial misallocation of money balances implies a discrepancy between income and spending and an associated trade imbalance. To characterize the preferred rate of adjustment of cash balances, we assume that spending by each country is proportional to money holdings.\(^1\) On the further assumption that velocities are equal in each country, $V = V^*$,\(^2\) world spending is equal to

\[ VM + eV M^* = V \bar{G}. \]

---

1 The assumption that spending is proportional to cash balances is only one of a number of possible specifications. Conditions for this expenditure function to be optimal are derived in Dornbusch and Mussa (1975). In general expenditure will depend on both income and cash balances.

2 In the long-run equilibrium higher $V$ than $V^*$ leaves us with a smaller share of the world money stock than foreigners but with nominal income shares in the two countries the same as when $V = V^*$.
For the tastes and technology specified in Part I, world spending on
domestically produced goods is given by

\[ V \tilde{g} \int_{0}^{\tilde{z}} b(z)dz = \theta(\omega)\tilde{V}g ; \quad \tilde{z} = A^{-1}(\omega). \]

In equilibrium, world spending on our goods must equal the value of our
full employment income, WL:

\[ WL = \theta(\omega)V\tilde{g} . \]

Equilibrium requires, too, that world spending on foreign goods equals
the value of foreign full-employment income:

\[ eW^*L^* = [1-\theta(\omega)]V\tilde{g} . \]

Equations (32) and (33) express what would seem to be the joint
determination of real and monetary variables. But, in fact, we could
have taken the short-cut of recognizing that the real equilibrium is
precisely that of the barter analysis developed in Part I. Dividing
(32) by (33) and substituting from (11) for the equilibrium relative
wage, we could use the latter equations as determining money wage
levels.

It is useful for purposes of our subsequent discussion of unemploy-
ment to analyze graphically the equilibrium determined by (32) and (33).
Figure 4 shows the full-employment market equilibrium schedules derived
from (32) and (33) for the domestic goods (labor) market, LL, and for
the foreign goods (labor) market, L*L.* The schedules are drawn for a

---

1 Actually LL is a plot of the implicit equation for W and W*, (32), written
as WL = \theta(W/eW*)V\tilde{g}. Likewise, L*L* is a plot of (33) with \omega there replaced
by W/eW*. For V \neq V*, Figure 4 holds only for long-run equilibrium.
given world money supply although the distribution of the money supply between countries need not be in the ultimate equilibrium proportions. The home country's market equilibrium schedule is positively sloped because an increase in domestic money wages creates an excess supply of domestic goods and labor. To avoid unemployment the increase in domestic wages has to be matched by a more than proportionate increase in foreign wages so as to reduce the relative price of domestic goods. The schedule is flatter than a ray through the origin because an equiproportionate increase in both wages leaves relative prices unchanged but raises the world price level, reduces the real money supply and therefore reduces spending relative to income. The resulting excess supply of domestic goods has to be eliminated by a decline in the domestic relative wage. The converse argument applies to the foreign country's market equilibrium schedule.

Figure 4 shows how the money wage, \( \tilde{W} \) and \( \tilde{W}^* \), are determined so as to establish full-employment market equilibrium in the goods and labor markets. The equilibrium money wages at point E obviously imply the equilibrium relative wage, given by the slope of a ray through the origin and point E.

The real and nominal equilibrium at point E in Figure 4 is independent of the distribution of the world quantity of money. The independence of the real equilibrium derives from the uniform homothetic tastes. The independence of the nominal equilibrium is implied by identical velocities. What does, however, depend on the short-run distribution of world money is the balance of payments. As in the work of Alexander (1952), we know this: when goods markets clear, the trade surplus or balance of payments, \( M \), of the home country is equal to the excess of income over spending, or:

\[
(34) \quad M = \tilde{W}L - VM.
\]

With the nominal wage independent of the distribution of world money, equation (34) therefore implies that the trade balance monotonically con-
verges to equilibrium at a rate proportional to the discrepancy from long-run equilibrium:

\[ M = V(M - \bar{M}) \; ; \; \bar{M} = \Theta(\bar{\omega})G. \]

The assumptions of this section make inoperative most of the traditional mechanisms discussed as part of the adjustment process: changes in the terms of trade, in home and/or foreign price levels, in relative prices of traded and nontraded goods (there being none of the latter), in double factorial terms of trade, or discrepancies in the price of the same commodity between countries. The features of the adjustment process of this section rely on (1) identical, constant expenditure velocities, (2) uniform-homothetic demand and (3) the absence of trade impediments. If velocities were constant but differed between countries, the level of money wages and prices, though not relative wages or prices, would depend on the world distribution of money. Relaxation of the uniform-homothetic taste assumption would make equilibrium relative prices a function of the distributions of spending. Finally, the presence of nontraded goods does provide valid justification for some of the behavior of relative prices and price levels frequently asserted in the literature; this behavior is studied in more detail in the next section.

1Suppose \( V > V^* \) and our share of the world money supply is initially larger than our equilibrium share. Then, as we lose \( M \), total world nominal income and nominal GNP falls. Always our share of nominal world GNP stays the same under the strong demand assumptions. Total world real output never changes during the transition; only regional consumption shares change. Therefore, both country's nominal price and wage levels fall in the transition, but such balanced changes have no real effects on either the transient or the final real equilibrium. A sudden permanent increase in \( V \) (with \( V^* \) staying unchanged) will instantly raise all world prices by less than the percentage increase in \( V \) (namely by \( \Theta(\bar{\omega}) \)). Now we have too much \( M \) and they too little \( M^* \), and the adjustment process just described will ensue.
C. The Price-Specie Flow Mechanism

In this section we discuss the adjustment process to monetary disequilibrium and enquire into the price effects associated with a redistributions of the world money supply when there are nontraded goods. Common versions of the Hume price-specie-flow mechanism usually involve the argument that in the adjustment process prices decline along with the money stock in the deficit country, while both rise in the surplus country. There is usually, too, an implication that the deficit country's terms of trade will necessarily worsen in the adjustment process and indeed have to do so if the adjustment is to be successful.

The preceding section demonstrated that the redistribution of money associated with monetary imbalance need have no effects on real variables (production, terms of trade, etc.) and on nominalg variables other than the money stock and spending. While this is clearly a very special case, it does serve as a benchmark since it establishes that the monetary adjustment process would be effective even in a one-commodity world.¹

To approach the traditional view of the adjustment process more closely, and to provide formal support for that view, we consider an extension to the monetary realm of our previous model involving nontraded goods. We return to the assumption that a fraction \((1-k)\) of spending in each country falls

¹Note that our analysis abstracts from possible adjustment lags in moving factors among industries. We should add, too, that our formulation is simplified in that we assume no asset alternative to money, and assume static expectations. Departure from either of these assumptions would of course modify the adjustment process under fixed rates. It would also have implications for flexible rates. Specifically one could think of an unanticipated change in the money stock being in the short run not entirely offset by depreciation with no real consumption effects. Instead it could induce some international lending and therefore effects analogous to those caused by a monetary disturbance under fixed rates.
on nontraded goods and accordingly equations (32) and (33) become:

\[(32') \quad WL = \theta(w)\bar{V}G + (1-k)\bar{\gamma}V\bar{G} \quad ; \quad \gamma = M/\bar{G}\]

\[(33') \quad eWL = [k - \theta(w)]\bar{V}G + (1-\gamma)(1-k)V\bar{G}.\]

These hold both in final equilibrium, and in transient equilibrium where specie is flowing.

Using this extended framework, we can draw on the analysis of the transfer problem in Part II to examine the adjustment that follows an initial distribution of world money between the two countries that differs from the long-run equilibrium distribution.

Suppose our \(M\) is initially excessive, say from a gold discovery here. Assume also that the gold discovery occurred when the world was in long-run equilibrium with the previous world money stock. As a result of our excess \(M\), we spend more than our earnings, incurring a balance of payments deficit equal to the rate at which our \(M\) is flowing out. In effect, the foreign economy is making us a real transfer to offset our deficit. As seen earlier, we, the deficit country, are devoting some of our excess spending on nontraded goods, shifting some of our resources to their production at the expense of our previous exports. We not only export fewer types of goods, but also import more types, and import more of each (\(\bar{w}\) rises and \(\bar{z}\) falls).

During the transition, while the real transfer corresponding to our deficit is taking place, our terms of trade are more favorable than in the long-run state. Our \(W\) is up, as is their \(W^*\), each relative to the old equilibrium; but our \(W\) is up relative to their \(W^*\). Therefore the price level of goods we continue to export is up relative to the price level of
goods they continue to export. The price level of our nontraded goods has also been raised relative to the price level of their nontraded goods, by the exact rise in \(W/W^*\). Relative to the old equilibrium, the price level of all goods we continue to produce (traded and nontraded) is up; the price level of all goods they continue to produce (traded and nontraded) is also up, but by a smaller proportion. Further, the price level here of goods we used to import will be relatively lower than in old equilibrium because \(W/W^*\) has fallen; the price level of goods we now import for the first time will also be relatively lower than in the old equilibrium. Abroad, the price level in local currency of the goods we continue to export will have risen relative to the old equilibrium.

Thus the price levels in the two countries have been changed differentially by the specie flow and implied real transfer. But that does not mean that any traded good ever sells for different prices in two places once conversion at the pegged exchange rate is reckoned in. In fact the divergence in weighted average (consumer) price levels is due to nontraded goods. The price level will rise in the gold-discovering country relative to the other country by precisely the share of nontraded goods in expenditure, \(1-k\), times the change in relative nontraded goods prices which in turn equals the terms of trade change.

It is a bit meaningless to say, "What accomplishes the adjustment is the relative movements of price levels for nontraded goods in the two countries", since we have seen that the adjustment will and can be made even when there are no such nontraded goods. It is meaningful to say, "The fact that people want to direct some of their expenditure to nontraded goods makes it necessary for resources to shift in and out of them
as a result of a real transfer, and such resource shiftings take place only because the terms of trade (double-factorial and for traded goods) do shift in the indicated way."

After specie has flowed out for some time, the surplus of our \( M \) will be less and the deficiency of their \( M^* \) will have been gradually corrected. The whole adjustment mechanism will turn itself off as the specie flow slows down to virtually nothing once the division of money stocks is again back at the normal fractions of world \( G \).

The adjustment process to a monetary disturbance is stable in the sense that the system converges to a long-run equilibrium distribution of money with balanced trade. To appreciate that point, we supplement equations (32') and (33') with (30) that continues to describe the monetary adjustment process. We note now, however, that \( W \) and \( W^* \) are endogenous variables whose levels in the short run depend on the distribution of the world money supply. Specifically the elasticity of a country's wages with respect to its nominal quantity of money, \( \delta \), is less than unity.¹ Accordingly, starting from full equilibrium, a redistribution of the world money supply toward the home country will create a deficit equal to:

\[
\frac{dM}{dM} = -V(1-\delta) \quad ; \quad 0 \leq \delta \leq 1
\]

¹The value of \( \delta \) can be calculated from equations (34), to be

\[
\delta \equiv (1-k) \frac{\gamma(1-\gamma)}{\gamma(1-\gamma) + \theta c} \text{where } c \text{ is the elasticity of the share of our traded goods in world spending, } c \equiv -\theta w/\theta > 0. \text{ If } A'(z) \text{ falls slowly, } c \text{ will be large.}
and accordingly establishes the stability of the adjustment process.

It is interesting to observe in this context that the presence of nontraded goods in fact -- contrary to James Laughlin's turn of the century worries -- slows down the adjustment process by comparison with a world of only traded goods. In the latter world, as we saw before, wages are independent of the distribution of money and accordingly $\delta = 0$. Further we observe that the speed of adjustment depends on the relative size of countries. The more equal countries are in terms of size the slower tends to be the adjustment process.

In concluding this section we note that nontraded goods or localized demand are essential to the correctness of traditional insistence that the adjustment process necessarily entails absolute and relative price and income movements. They are, of course, in no way essential to the existence of a stable adjustment process, nor is there at any time a need for a discrepancy of prices of the same commodity across countries in either case.\(^1\)

A final remark concerns the adjustment to real disturbances such as demand shifts or technical progress. It is certainly true that, whether the exchange rate is fixed or flexible, real adjustment will have to take place and cannot be avoided by choice of an exchange rate regime. So long as wages and prices are flexible, it is quite false to think that fixed parities "put the whole economy through the rings of adjustment" while in

\(^1\)The Ricardian technology is special in that there can be no range of goods both imported and produced at home. Therefore, the cross elasticity of supply between nontraded goods and exports must be greater than the zero cross elasticity between nontraded goods and imports. Consequently, a transfer must shift the terms of trade (for goods and factors) in the stated orthodox way, favorably for the receiver. Once we leave the Ricardian technology, they may move in the opposite direction -- showing that it is not necessary for stability of adjustment that nontraded goods fall in price in the specie-losing country.
floating rate regimes "only the export and import industries make the adjustment". It is true, however, that once we depart from flexible wages and prices there may well be a preference for one exchange rate regime over another. The next section is devoted to that question.
D. Sticky Wages

The last question we address in this part concerns the implications of sticky money wages. For a given world money supply, downward stickiness of money wages implies the possibility of unemployment. (We assume upward flexibility of wages once full employment is attained.)

Consider first the case of fixed exchange rates. Denote actual employment levels in each country by \( \bar{L} \) and \( \bar{L}^* \) as against full employment levels \( L \) and \( L^* \). With the latter substitution in (32) and (33) we must interpret the market equilibrium schedules in Figure 4 as full employment loci for the two countries. For any new \((\bar{L}, \bar{L}^*)\) pair in those equations we can superimpose in Figure 4 two new intersecting schedules. For \( \bar{L} < L \) the new \( \bar{L}\bar{L} \) schedule lies above and to the left of the \( LL \) schedule. For \( \bar{L}^* < L^* \) the new \( \bar{L}^*\bar{L}^* \) schedule lies to the right and below the \( L^*L^* \) schedule.

An increase in foreign money wage rates, starting from a full-employment equilibrium at point \( E \) in Figure 4, gives rise to a new equilibrium at point \( E' \) with unemployment abroad, and an improvement in the foreign relative wage and terms of trade. In the home country employment remains full and the excess demand for domestic goods and labor causes the domestic money wage rate to rise and thus partly offset the terms of trade deterioration. The increase in the equilibrium domestic nominal wage using (32) and (33) is equal to

\[
(36) \quad \hat{W} = \frac{\varepsilon}{1 + \varepsilon} \hat{W}^* \quad ; \quad \varepsilon \equiv -\theta' \omega \theta > 0
\]

where \( \varepsilon \) is the elasticity of the share of spending that falls on domestic goods. The proportional change in foreign employment is given by
and is greater in absolute value the smaller the foreign country as measured by its share in world income and spending.

Consider now the same foreign money wage increase under flexible exchange rates. In this case foreign employment declines in proportion to the wage increase and the home country's exchange rate appreciates. The appreciation is equal to:

\[
\hat{e} = -\frac{e}{1+\epsilon}[1-\theta]\frac{W}{W^*}
\]

and therefore does not fully offset the foreign wage increase. It follows that the domestic relative wage declines and the home country's terms of trade deteriorate. The extent of the terms of trade deterioration is larger, the smaller the home country as measured by the share in world income.

A comparison of the effects of a wage increase, without a corresponding increase in money, under fixed and flexible rates reveals that the fixed rate system implies larger changes in the terms of trade, employment and trade patterns in comparison with a flexible rate world. That is because the wage rate increase is partially offset in the flexible rate regime by an exchange rate depreciation.

---

1Under flexible rates and inflexible wages foreign employment is given by \( \hat{L} = \frac{M^*V^*/W^*}{W} \). The equilibrium relative wage is obtained from equilibrium in the trade balance \( \omega = (\hat{L}/\hat{L})\theta/(1-\theta) \). Finally the exchange rate can be recovered from the definition of the relative wage \( \omega = W/eW^* \).
The previous example considered an initial position of full employment equilibrium. We will now use the diagrammatic apparatus of Figure 4 to examine the effects of a foreign wage increase, starting from worldwide unemployment equilibrium in which wage levels are too high everywhere to sustain full employment. Figure 5 shows an initial equilibrium (with unemployment) at point E". (The full employment equilibrium point E of Figure 4 is included in Figure 5, but the full employment schedules LL and L * L * are omitted for clarity.) Corresponding to the unemployment equilibrium at E" are LL̂ and L̂ L̂ employment loci for the two countries passing through that point, and indicating the employment levels in the two countries. Also corresponding to point E" are the nominal wage levels in the two countries, Ŵ and W*, that are too high to sustain full employment in either country.

Now consider an increase in the foreign wage rate from Ŵ to W*. With our wage rate remaining at Ŵ, the new (unemployment) equilibrium will be at point E in Figure 5. Passing through E are the employment loci LL and L * L*; thus the new equilibrium has our employment level higher than it was at E" before the increase in the foreign wage rate, and has foreigners' employment level lower than it was at E". The foreigners' rise in wages has increased their costs of production and prices of goods, and thus shifted demand towards our goods. With our money wages and prices unresponsive because of unemployment, the increased demand for our goods is met entirely by increased employment and production here. Our expansion in employment is accompanied by a deterioration in our terms of trade by the full extent of the foreign wage increase. In the new (underemployment) equilibrium, we produce some goods that used to be produced by foreigners. World nominal income (equal to VG) is unchanged by
the increase in the foreign wage rate; our nominal income has risen (since the nominal wage is unchanged and employment has increased) and foreign nominal income has fallen (their employment level falls proportionately more than their nominal wage rises as indicated by (37)). Foreigners who continue to be employed are better off than they were in the old equilibrium at E" because the prices of goods they produce have risen in the same proportion as their wages, while other goods are now relatively cheaper. By the same token, our laborers who were previously employed are worse off.

In the new equilibrium we have more money than we did in the old equilibrium, since our share of the world money stock is proportional to our share of world income. The transition process to the new (underemployment) equilibrium is thus one in which foreigners run a balance of payments deficit. Under our assumption that factors can be reallocated instantaneously between industries, the transition process is quite simple. Simultaneously with the increase in the foreign wage rate, world production patterns take on their new steady state configuration. Consumption rates, however, do not adjust instantly to the new steady state values. Instead, foreigners initially spend at the same rate (in nominal terms) that they did in the old equilibrium at E"; and since their nominal income has fallen, they are running down their cash balances. Their consumption gradually falls to its new equilibrium level and ours correspondingly rises from the old level towards the new equilibrium level as the world's stock of money is redistributed towards us.

Under flexible exchange rates, by contrast, an increase in the foreign wage has no effects on our employment, while foreign employment is reduced by their wage increase. Our terms of trade worsen, as in the full employment
case, with the exchange rate appreciation falling short of the foreign wage increase, as demonstrated in (38). The foreign wage increase under flexible exchange rates reduces their employment level by less than it is reduced under fixed rates, because the exchange rate change absorbs some of its impact.
Appendix I

Historical Remark

Figure 1 seems to be new. Elliot (1950) gives a somewhat different diagram, one that makes explicit the meaning of Marshall's 1879 "bales" (which, by the way, happen to work only in the 2-country constant-labor-costs case). In terms of the present notations, Elliott plots for the U.S. offer curve the following successive points traced out for all \( w \) on the range \([0, \infty)\): on the vertical axis is plotted our total real imports valued in foreign labor units ("our demand for bales of their labor", so to speak), namely

\[
\int_{\tilde{z}}^{\infty} \left[ \frac{P^*(z)/W}{C^*} \right] C(z) dz = \int_{\tilde{z}}^{\infty} a^*(z) C(z) dz;
\]

and on the horizontal axis, our total real exports valued in home labor units ("our supply of bales of labor to them"), namely

\[
\int_{0}^{\tilde{z}} \left[ \frac{P(z)}{W} \right] \left[ Q(z) - C(z) \right] dz = \tilde{L} - \int_{0}^{\tilde{z}} a(z) C(z) dz.
\]

It is to be understood that \( \tilde{z} \) is a function of \( w \), namely the inverse function \( A^{-1}(w) \); also that \( C(z) \) are the amounts demanded as a function of our real income \( L \) and the \( P(z)/W \) function defined for each, namely as

\[ \text{Min} [w a(z), a^*(z)]. \]

Because we have a continuum of goods, we avoid Elliott's branches of the offer curve that are segments of various rays through the origin. The reader will discern by symmetry considerations how the foreign offer curve is plotted in the same \((L, L^*)\) quadrant, by varying \( w \) to generate
the respective coordinates

\[
\left[ \begin{array}{c}
\int_0^1 \hat{a}(z)C^*(z) \, dz, L - \int_0^1 \hat{a}(z) \hat{C}(z) \, dz
\end{array} \right].
\]

Our model will require the Elliott-Marshall diagram to generate a unique solution under uniform-homothetic demand. Unlike our Figure 1, the Elliott diagram can handle the general case of non-homothetic demands in the two countries; but then, as is well known, multiple solutions are possible, some locally stable and some unstable. The price one pays for this generality is that, as Edgeworth observed, the Marshallian curves are the end-products of much implicit theorizing, with much that is interesting having taken place offstage.
Annotated Bibliography


Mill, J. S. Principles of Political Economy (1848). This reproduces relevant parts of Mill's 1829 work published in his 1844 Essays on Some Unsettled Questions of Political Economy.


